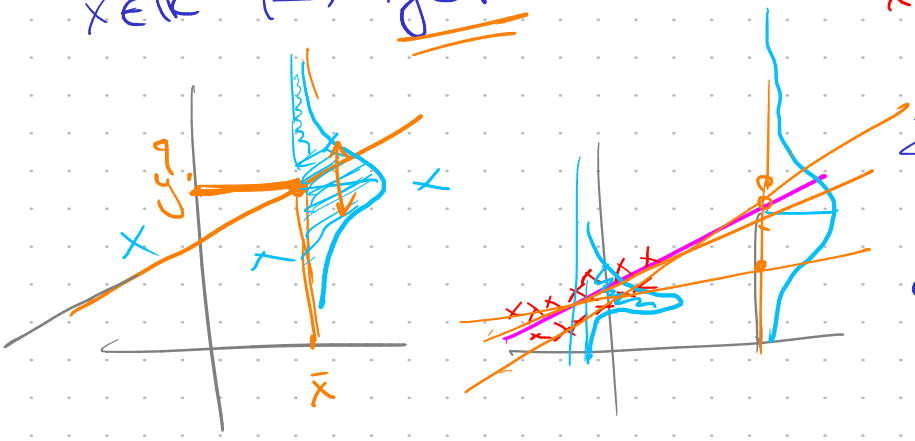
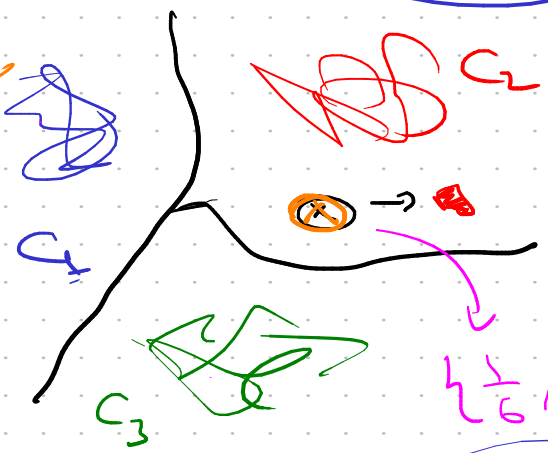
Regression

$\bar{x} \in \mathbb{R}^d \mapsto y \in \mathbb{R}$



classification

$\bar{x} \in \mathbb{R}^d \mapsto y \in \{C_1, C_2, C_3\}$



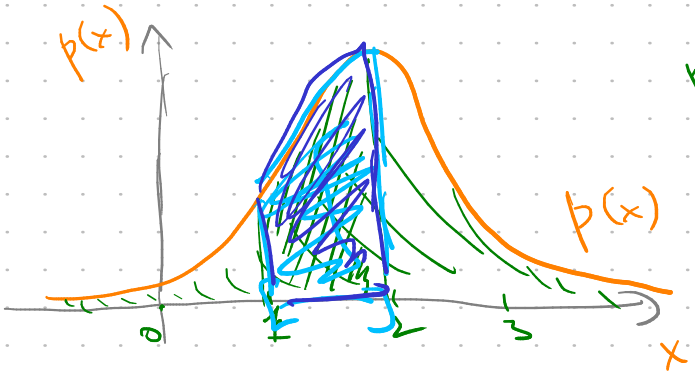
$y \in \{C_1, C_2, C_3\}$

$p_i = p(y=C_1),\ p(y=C_2),\ p(y=C_3)$

$\forall i \quad p_i \geq 0, \quad \sum_i p_i = 1$

$x \in (-\infty, \infty)$

$p(x=a) = 0$



$p\left(x \in [1,2]\right) = \int_1^2 p(x)\,dx$

$\int_{-\infty}^{\infty} p(x)\,dx = 1$

joint probability

$p(x,y) \qquad p(x=a,\ y=b)$



$p(x_1 = \square,\ x_2 = \text{même}) = 1/12$

$p(x,y) = p(x) \cdot p(y)$



$p(x = \square) = 1/6$

$p(x \text{ même}) = 1/2$

$p(x = \square,\ x \text{ même}) = 1/6$

$p(x = \square,\ x \text{ nemême}) = 0$

conditional probability

$$p(x|y) = \frac{p(x,y)}{p(y)}$$

$$p(x = \boxdot \mid x \text{ четное}) = \frac{1/6}{1/2} = 1/3$$

$p(\bar{\theta} \mid D)$ — параметр, данные

Coin tossing

$D = t\,t\,t\,h\,t\,h$

$\theta = p(\text{„орел“} \mid \theta)$

$1-\theta = p(\text{„решка“} \mid \theta)$

model

$\theta \in [0,1]$

Likelihood / правдоподобие

$p(D|\theta) =$ $\quad D = \{d_1, d_2, \dots, d_N\}$

$= p(d_1|\theta)\, p(d_2|\theta) \dots p(d_N|\theta)$

$p(D|\theta) = (1-\theta)(1-\theta)(1-\theta)\theta(1-\theta)\theta = \theta^2(1-\theta)^4$

$p(D = \{k \text{ орлов}, m \text{ решек}\} \mid \theta) = \theta^k (1-\theta)^m$ — правдоподобие

$p(D|\theta) \xrightarrow{\theta} \max$ $\quad \theta_{ML}$ maximum likelihood

$k\cdot\theta^{k-1}(1-\theta)^m - m\theta^k(1-\theta)^{m-1} = 0$

$\theta^{k-1}(1-\theta)^{m-1}\big(k(1-\theta) - m\theta\big) = 0$

$\theta = 0, \quad \theta = 1, \quad k - k\theta - m\theta = 0$

$$\theta_{ML} = \frac{k}{k+m}$$

$p(y|x)p(x) = p(x,y) = p(x|y)p(y)$

$$p(y|x) = \frac{p(x|y)p(y)}{p(x)}$$

Bayes' rule formula theorem

posterior: $p(\bar{\theta}|D)$ = likelihood $p(D|\bar{\theta})$ prior $p(\bar{\theta})$ / $p(D)$ $\propto p(D|\bar{\theta})p(\bar{\theta})$

$p(D) = \int p(D|\bar{\theta})p(\bar{\theta})\,d\bar{\theta}$

Bayes formula

t: test
d: disease

"likelihood" $p(t|d)$

$p(t=1|d=1) = 0.95$
$p(t=0|d=1) = 0.05$
$p(t=1|d=0) = 0.05$
$p(t=0|d=0) = 0.95$

prior

$$p(d=1|t=1) = \frac{p(t=1|d=1)\,p(d=1)}{p(t=1|d=1)\,p(d=1) + p(t=1|d=0)\,p(d=0)}$$

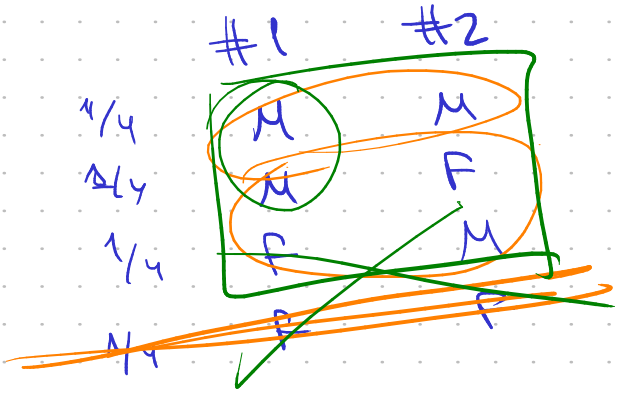
$p(t=1, d=1)$

"data"

posterior

$$= \frac{0,95 \cdot 0,01}{0,95 \cdot 0,01 + 0,05 \cdot 0,99} \approx \frac{1}{6}$$

$\approx 1\%$; $0,95 \cdot 0,01 \approx 1\%$; $0,05 \cdot 0,99 \approx 5\%$



- M M
- M F
- F M
- F F

$p(\exists F | \exists M) = \frac{2}{3}$

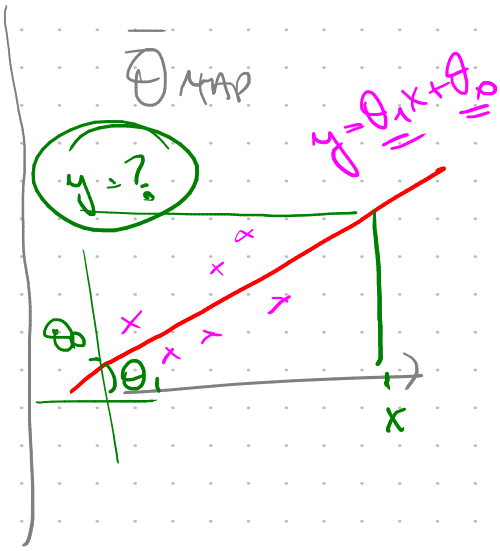
M M
M F
F M
F F

$p(\text{больной} \mid \text{кровь}) = ?$

$p(\text{кровь} \mid \text{нездоров}) = 0.01$

$\Downarrow$

$p(\text{здоров} \mid \text{нездоров}) = 0.99$



predictive distribution

$$p(y \mid x, D) = \int \underbrace{p(y \mid x, \theta)\, p(\theta \mid D)}_{p(y, \theta \mid x, D)} d\theta =$$

$D = h$

$p(D \mid \theta) = \theta \xrightarrow{\theta \to \max}$

$\theta_{ML} = 1$

$D = t$

$\theta_{ML} = 0$

$$\underbrace{p(\theta \mid D)}_{\text{posterior}} \propto \overbrace{p(D \mid \theta)}^{\text{likelihood}} \cdot \underbrace{p(\theta)}_{\text{prior}}$$

