① Bayes rule

$$\underbrace{p(\bar\theta \mid D)}_{\text{posterior}} = \frac{\overbrace{p(\bar\theta)}^{\text{prior}}\,\overbrace{p(D\mid\bar\theta)}^{\text{likelihood}}}{\underbrace{p(D)}_{\text{evidence}}}$$

$$p(D) = \int p(\bar\theta)\,p(D\mid\bar\theta)\,d\bar\theta$$

$$p(D\mid\bar\theta) = \prod_{n=1}^{N} p(d_n\mid\bar\theta)$$

1) $p(D\mid\bar\theta) \xrightarrow{\bar\theta} \max$ $\qquad \bar\theta_{ML} = \arg\max_{\bar\theta} p(D\mid\bar\theta)$

maximum likelihood

$$\log p(D\mid\bar\theta) = \sum_n \log p(d_n\mid\bar\theta)$$

2) $p(\bar\theta\mid D) \xrightarrow{\bar\theta} \max$ $\qquad \bar\theta_{MAP} = \arg\max_{\bar\theta} p(\bar\theta\mid D)$

maximum a posteriori

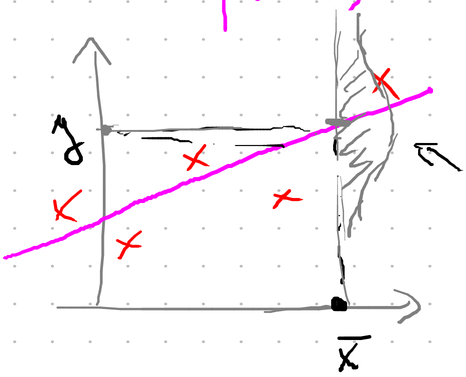
regularizer

$$\log p(\bar\theta\mid D) = \sum_n \log p(d_n\mid\bar\theta) + \log p(\bar\theta) + \text{Const}$$

3) predictive distribution

$$p(\bar x\mid D) = \int p(\bar x, \bar\theta \mid D)\,d\bar\theta = \int \underbrace{p(\bar x\mid \bar\theta, \cancel{D})}_{\text{likelihood}}\,\underbrace{p(\bar\theta\mid D)}_{\text{posterior}}\,d\bar\theta =$$

$$= \mathbb{E}_{p(\bar\theta\mid D)}\left[p(\bar x\mid\bar\theta)\right]$$



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p(кровь | невиновен) = 1%

p(невин. | кровь) = 1%

p(убийц | дом. нас.) = 1/2500

p(убил муж | дом. нас. & убита) = 7/8 $\qquad$ 6/7

② Bernoulli trials

$\theta = p(\text{орёл})$, $D = h\,t\,h\,h\,t\,h\,h$

$$p(D|\theta) = \theta(1-\theta)\ldots = \underline{\theta^n \cdot (1-\theta)^m}$$

$$p(D|\theta) \xrightarrow{\theta} \max$$

$$\frac{\partial p(D|\theta)}{\partial\theta} = n\theta^{n-1}(1-\theta)^m - m\theta^n(1-\theta)^{m-1} = 0$$

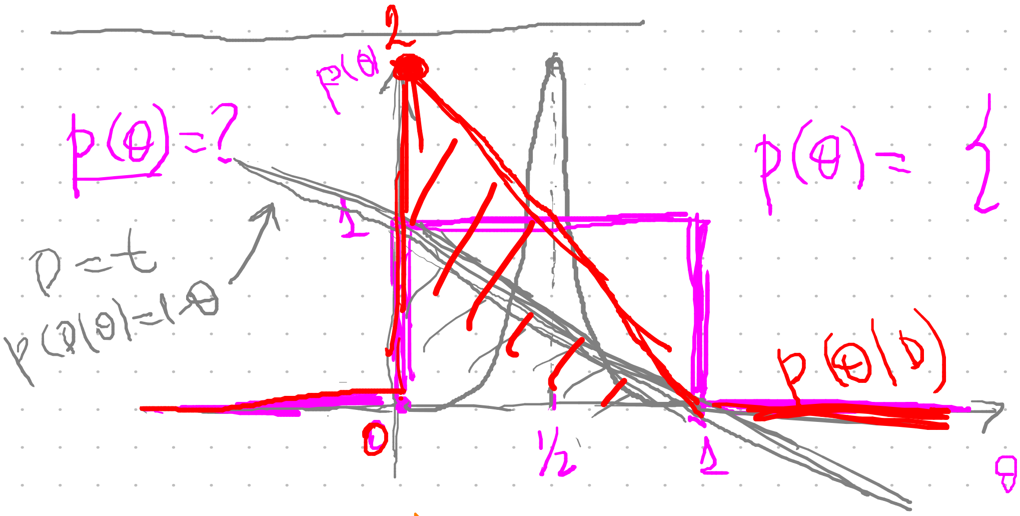
$$\theta^{n-1}(1-\theta)^{m-1}\big(n(1-\theta) - m\theta\big) = 0$$

$$\theta = 0,\ 1,\ \frac{n}{n+m}$$

$$\boxed{\theta_{ML} = \frac{n}{n+m}}$$

$\underline{D = t} \not\Rightarrow \underline{\theta_{ML} = 0}$ $\qquad$ $\underline{p(t|\theta) = 1-\theta}$

$\underline{\theta_{ML} = -\infty}$



$$p(\theta) = \begin{cases} 1, & \theta \in [0,1] \\ 0, & \text{—"—} \end{cases}$$

$$p(D|\theta) = \theta^n(1-\theta)^m$$

$$p(\theta|D) = \begin{cases} \frac{1}{p(D)}\theta^n(1-\theta)^m, & \theta \in [0,1] \\ 0, & \text{—"—} \end{cases}$$

$$p(D) = \int_0^1 \theta^n(1-\theta)^m\,d\theta = B(n+1, m+1) =$$

$$= \frac{\Gamma(n+1)\,\Gamma(m+1)}{\Gamma(n+m+2)} = \frac{n!\,m!}{(n+m+1)!}$$

$$p(\theta|D) = \frac{(n+m+1)!}{n!\,m!}\,\underline{\theta^n(1-\theta)^m}, \quad \underline{\theta \in [0,1]}$$

$\max_{\theta}$

$$\boxed{\theta_{MAP} = \frac{n}{n+m}}$$

$$p(\text{open}|D) = \int \overbrace{p(\text{open}|\theta)}^{\theta}\,\overbrace{p(\theta|D)}\,d\theta =$$

$$= \int_0^1 \theta \cdot \frac{(n+m+1)!}{n!\,m!}\,\theta^n(1-\theta)^m\,d\theta = \ldots \int_0^1 \theta^{n+1}(1-\theta)^m\,d\theta$$

$$= \frac{\cancel{(n+m+1)!}}{\cancel{n!\,m!}} \cdot \frac{(n+1)\cancel{!}\,\cancel{m!}}{(n+m+2)\cancel{!}}$$

Bayesian smoothing

$$\boxed{p(\text{open}|D) = \frac{n+1}{n+m+2}}$$

Laplace's rule of succession