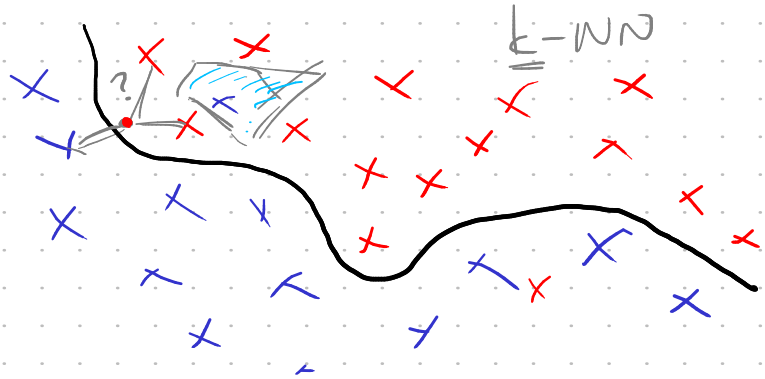
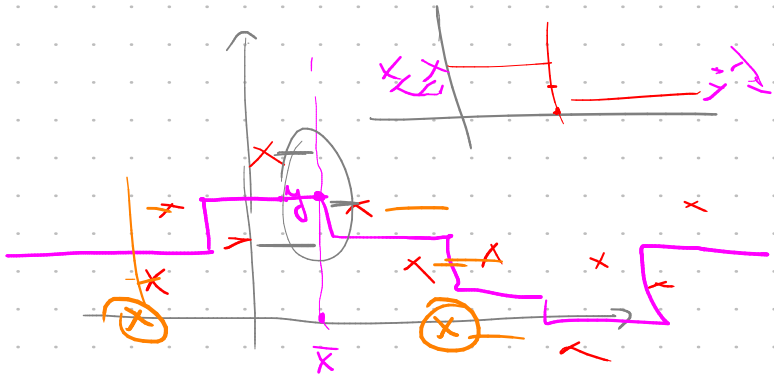
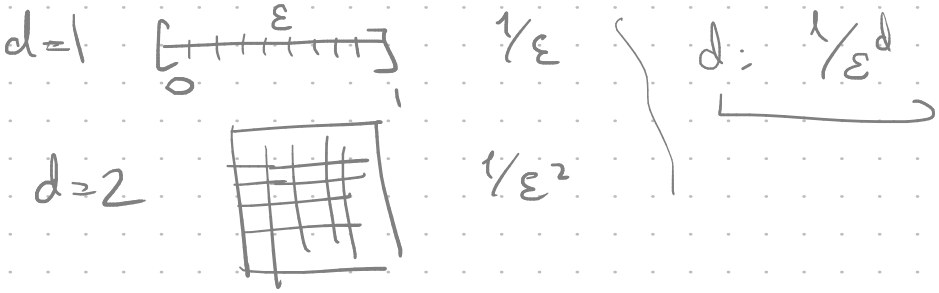
(1) Nearest neighbors





(2) Curse of dimensionality

— численное интегрирование

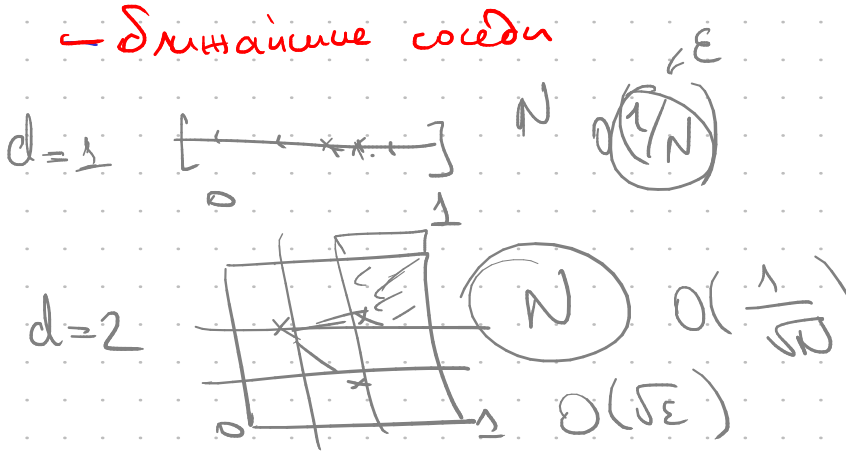


$$p(\bar{x}|D) = \int p(\bar{x}|\bar{\theta}) p(\bar{\theta}|D) d\bar{\theta} =$$

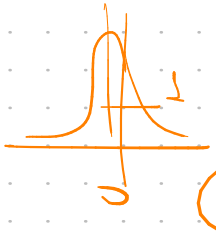
$$= \mathbb{E}_{p(\bar{\theta}|D)}[p(\bar{x}|\bar{\theta})] \approx \frac{1}{R} \sum - -$$

$$\bar{\theta} \sim p(\bar{\theta}|D)$$

— ближайшие соседи



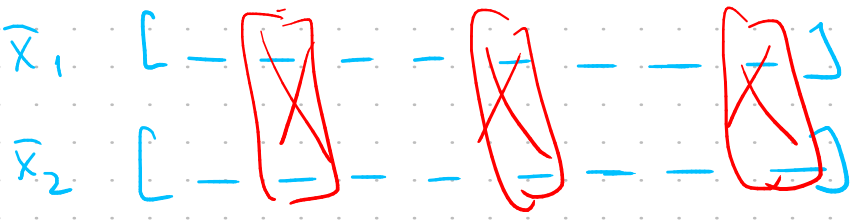
$O(\varepsilon^{1/d})$



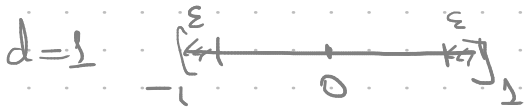
$x_i \sim N(0,1)$

$d^2 = \sum x_i^2$

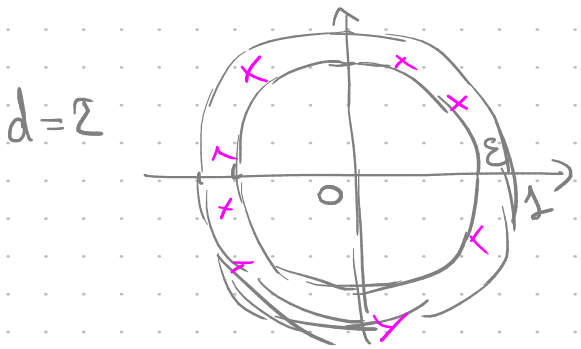
$\bar{x} \sim N(\bar{0}, \bar{I})$



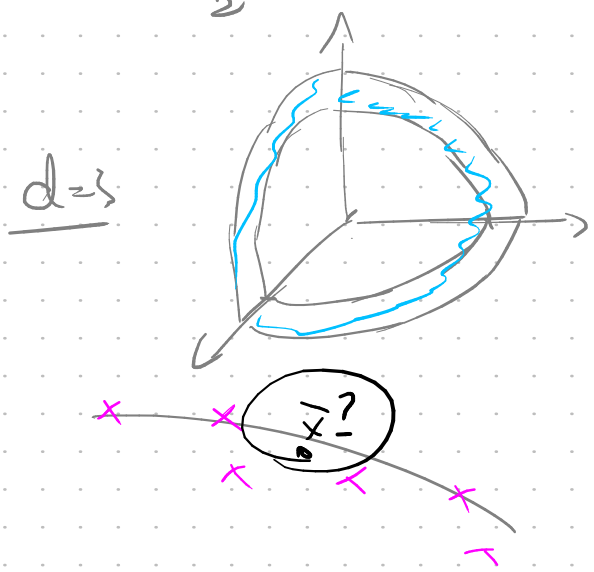
- доверительная область



ε



$$\frac{1^2-(1-\varepsilon)^2}{1^2} = 2\varepsilon - \varepsilon^2$$



$$1-(1-\varepsilon)^3$$

$$1-(1-\varepsilon)^d \to 1$$

(3) Statistical decision theory

$\bar{x} \in \mathbb{R}^d$, $y \in \mathbb{R}$ $\quad D = \{(\bar{x}_n, y_n)\}_{n=1}^N$ $\quad f: \bar{x} \mapsto y$

$$L(y, \underbrace{f(\bar{x})}_{\hat{y}}) = (y - f(\bar{x}))^2$$

$p(\bar{x}, y) = p(\bar{x})\, p(y|\bar{x})$

Expected prediction error

$$EPE[f] = E_{p(\bar{x},y)}[(y-f(\bar{x}))^2] \xrightarrow{f} \min$$

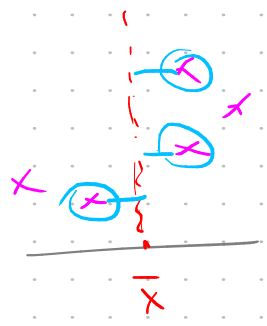
$$= \int (y-f(\bar{x}))^2\, p(y|\bar{x})\, p(\bar{x})\, dy\, d\bar{x} =$$

$$= \int \left(\int (y-f(\bar{x}))^2\, p(y|\bar{x})\, dy \right) p(\bar{x})\, d\bar{x} \xrightarrow{f} \min$$

$f(\bar{x}) \to \min$

$$\hat{f}(\bar{x}) = E_{p(y|\bar{x})}[y]$$

regression function



$$\hat{f}(\bar{x}) = \mathbb{E}_{p(y|\bar{x})}[y] \overset{\text{good!}}{\approx} \frac{1}{R}\sum_{z=1}^{R} y_z \overset{\text{bad!}}{\approx} \frac{1}{R} \sum_{x_z \in R\text{-}NN(x)} y_z$$

$y_z \sim p(y|\bar{x})$

$y \in \{1, \dots, K\}$

$f: \bar{x} \mapsto y$

$$L(y, f(\bar{x})) = \begin{cases} 0, & y = f(\bar{x}) \\ 1, & y \neq f(\bar{x}) \end{cases}$$

$$EPE[f] = \int\int L(y, f(\bar{x}))\, p(y|\bar{x})\, p(\bar{x})\, dy\, d\bar{x} =$$

$$= \int \left(\sum_{k=1}^{K} L(C_k, f(\bar{x}))\, p(C_k|\bar{x}) \right) p(\bar{x})\, d\bar{x}$$

$$\hat{f}(\bar{x}) = \underset{k}{\operatorname{argmax}}\, p(C_k|\bar{x})$$

$f(\bar{x}) \to \min$

$f(\bar{x})$ \ y	C_1	C_2
C_1	0	1000
C_2	1	0

(Spalte: y = wahre Klasse; Zeile: $f(\bar{x})$ = geschätzte Klasse)

$$\hat{f}(\bar{x}) = \underset{f(\bar{x})}{\operatorname{argmin}} \sum_{k=1}^{K} L(C_k, f(\bar{x}))\, p(C_k|\bar{x})$$

(4) Bias-variance-noise decomposition

$y \in \mathbb{R}$,

$$EPE[f] = \iint (y - f(\bar{x}))^2\, p(\bar{x}, y)\, dy\, d\bar{x} = \qquad \hat{f}(\bar{x}) = \mathbb{E}_{p(y|\bar{x})}[y]$$

$$(y - \hat{f} + \hat{f} - f)^2$$

Noise

$$= \int (y - \hat{f}(\bar{x}))^2 p(\bar{x}, y)\, dy\, d\bar{x} + \int (y - \hat{f}(\bar{x}))(\hat{f}(\bar{x}) - f(\bar{x}))\, p(\bar{x}, y)\, dy\, d\bar{x}$$

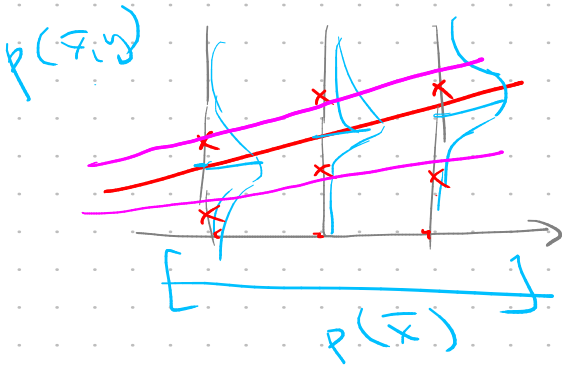
$p(y|\bar{x})p(\bar{x})$

$$\int \left(\int (y - \hat{f}(\bar{x}))\, p(y|\bar{x})\, dy \right) (\hat{f} - f)\, p(\bar{x})\, d\bar{x}$$

$\int (z - \mathbb{E}[z])\, p(z)\, dz = 0$

$$+ \int (\hat{f}(\bar{x}) - f(\bar{x}))^2\, p(\bar{x})\, d\bar{x}$$

$$\int \left(\hat{f}(\bar{x}) - f(\bar{x};D)\right)^2 p(\bar{x},y)\,dy\,d\bar{x} =$$



$$E_D[f(\bar{x};D)] \qquad D \sim p(\bar{x},y)$$

$$\left(\hat{f} - E_D f + E_D f - f\right)^2$$

$$= \int \left(\hat{f}(\bar{x}) - E_D f(\bar{x};D)\right)^2 p(\bar{x},y)\,dy\,d\bar{x} + \int \left(\hat{f} - E_D f\right)\left(E_D f - f\right) p(\bar{x},y)\,dy\,d\bar{x}$$

$E_D f - f(\bar{x};D)$

$$+ \int \left(E_D f(\bar{x},D) - f(\bar{x};D)\right)^2 p(\bar{x},y)\,dy\,d\bar{x}$$

$$\int \left(E_D f - f\right) p(\bar{x})\,d\bar{x} = 0$$

$$EPE[f] = \int \left(\hat{f}(\bar{x}) - E_D f(\bar{x};D)\right)^2 p(\bar{x})\,d\bar{x} \qquad \text{Bias}^2$$

$$+ \int \left(E_D f(\bar{x};D) - f(\bar{x};D)\right)^2 p(\bar{x})\,d\bar{x} \qquad \text{Variance}$$

$$+ \int \left(y - \hat{f}(\bar{x})\right)^2 p(\bar{x},y)\,d\bar{x}\,dy \qquad \text{Noise}$$