① Variational autoencoders

$D = \{\bar{x}_n\}_{n=1}^{N}$

$\bar{x}$ — Enc — $\bar{z}$ — Dec — $\hat{\bar{x}}$

$\|\bar{x} - \hat{\bar{x}}\|^2$

Reconstruction loss

$\|VGG(\bar{x}) - VGG(\hat{\bar{x}})\|^2$

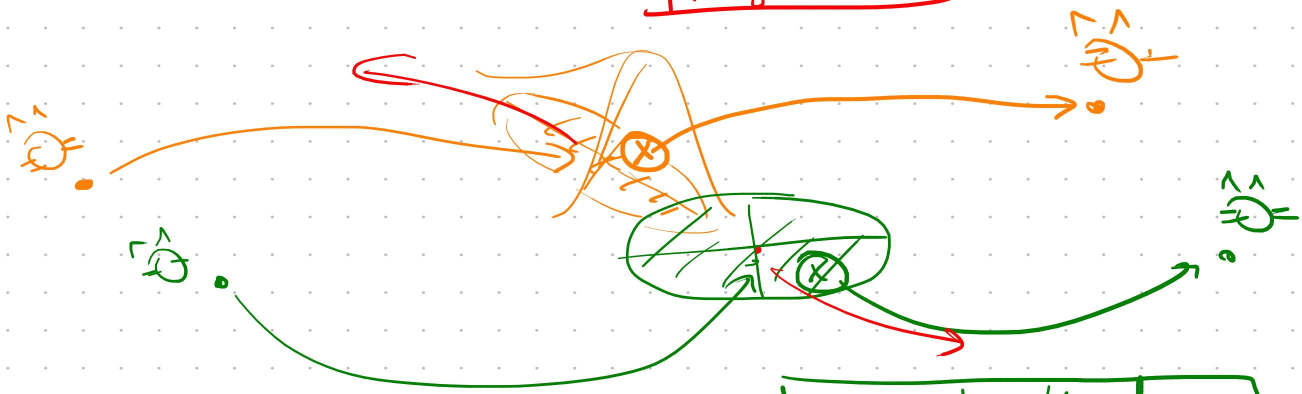
AAE

0/1 — Discr — $\bar{z}_{fake}$, $\bar{z}_{real}$

$\bar{x}$ — Enc — $\cancel{\bar{z}}$ — Dec — $\hat{x}$

$\bar{x}$ — Enc — $p(\bar{z}|\bar{x}) \sim \bar{z}$ —— $\hat{x}$

Regularizer



Reconstruction loss

$q(\bar{z})$

$\bar{x}$ — Enc — $\bar{\mu}_x$, $\bar{\sigma}_x = \begin{pmatrix} \sigma_1 & & 0 \\ & \ddots & \\ 0 & & \sigma_d \end{pmatrix}$ — $\mathcal{N}(\bar{z}|\bar{\mu}_x, \bar{\sigma}_x) \sim \bar{z}$ — Dec — $\hat{x}$

$\approx p(\bar{z}|\bar{x})$

Regularizer

$\mathcal{N}(\bar{z}|\bar{\mu}_x, \bar{\sigma}_x)$

$\approx \mathcal{N}(\bar{z}|\bar{0}, \bar{I})$

② VAE formally

simple „decoder" „encoder"

$$\underbrace{p(\bar z)\; p(\bar x|\bar z)}_{\text{generation}} = p(\bar x, \bar z) = p(\bar x)\; p(\bar z|\bar x)$$

$$\mathcal{N}(\bar z|\bar 0, I)\cdot \mathcal{N}(\bar x|\mathrm{Dec}(\bar z), c\cdot I) = \quad \cdot\; p(\bar z|\bar x)$$

variational approx. z^2 $q(\bar z)$

$$q(\bar z) = \mathcal{N}(\bar z|\bar\mu_x, \bar\sigma_x)$$

$$\log p(\bar x, \bar z) = \log\big(p(\bar x)\, p(\bar z|\bar x)\big)$$

$$\log p(\bar x) = \log p(\bar x, \bar z) - \log p(\bar z|\bar x) \quad \Big|\, \mathbb{E}_{q(\bar z)}$$

$$\log p(\bar x) = \mathbb{E}_{q(\bar z)}\big[\log p(\bar x, \bar z) - \log p(\bar z|\bar x)\big] \pm \mathbb{E}_{q(\bar z)}\big[\log q(\bar z)\big]$$

$$\log p(\bar x) = \underbrace{\mathbb{E}_{q(\bar z)}\Big[\log \frac{p(\bar x, \bar z)}{q(\bar z)}\Big]}_{\mathcal{L}(q)} + \mathbb{E}_{q(\bar z)}\Big[\log \frac{q(\bar z)}{\log p(\bar z|\bar x)}\Big]$$

$$\text{Const} = \mathcal{L}(q) + KL\big(q(\bar z)\,\|\, p(\bar z|\bar x)\big)$$

$\mathcal{L}(q)$: q → max

KL: q → min

variational lower bound

evidence – ELBO

$$KL(q\|p) = \int q \cdot \log \frac{q}{p}\, d\bar z$$

$$\mathcal{L}(q) = \int q(\bar z) \cdot \log \frac{p(\bar x, \bar z)}{q(\bar z)}\, d\bar z =$$

$$= \int q(\bar z) \underbrace{\log p(\bar x, \bar z)}_{\log p(\bar z) + \log p(\bar x|\bar z)} d\bar z - \int q(\bar z) \log q(\bar z)\, d\bar z =$$

$$= \mathbb{E}_{q(\bar z)}[\log p(\bar x|\bar z)] + \mathbb{E}_q[\log p(\bar z)] - \mathbb{E}_q[\log q(\bar z)]$$

$$= \mathbb{E}_{q(\bar z)}[\log p(\bar x|\bar z)] - \underbrace{\mathbb{E}_{q(\bar z)}\left[\log \frac{q(\bar z)}{p(\bar z)}\right]}$$

$$= \mathbb{E}_{q(\bar z)}[\log \mathcal{N}(\bar x|\mathrm{Dec}(\bar z), c\cdot I)]$$

$$= \mathbb{E}_{q(\bar z)}\left[\mathrm{Const} - \frac{1}{2c^d}\cdot\|\bar x - \mathrm{Dec}(\bar z)\|^2\right]$$

$$= \mathrm{Const} - \underbrace{\mathrm{Const}\cdot\mathbb{E}_q[\|\bar x - \mathrm{Dec}(\bar z)\|^2]}_{L_{recon}}$$

$$L_{recon} = \frac{1}{N}\sum_n \|\bar x_n - \mathrm{Dec}(\mathrm{Enc}(\bar x_n))\|^2$$

$$= KL(q(\bar z)\,\|\,p(\bar z)) \qquad p(\bar z) = \mathcal{N}(\bar z|\bar 0, I)$$

$$= \underbrace{KL(\mathcal{N}(\bar z|\bar\mu_x,\bar\sigma_x)\,\|\,\mathcal{N}(\bar z|\bar 0, I))}_{L_{reg}}$$

$$h(q) = \mathrm{Const} - (\mathrm{Const}\cdot L_{recon} + L_{reg})$$

$h(q)$: ↓ max

L_{recon}, L_{reg}: ↓ q min

③ Reparametrization trick

$$\mathcal{N}(\bar z|\bar 0, I) \odot \bar\sigma_x + \bar\mu_x$$

$$= \mathcal{N}(\bar z|\bar\mu_x,\bar\sigma_x) \sim \bar z$$