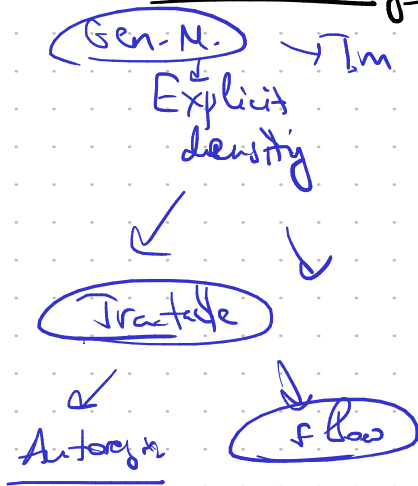


Normalizing flows



$$\bar{X} = (x_1, x_2, \dots, x_n)$$

$$p(x_i | x_1, \dots, x_{i-1})$$



$$p(\bar{x})$$

$$p(\bar{z}) = \mathcal{N}(\bar{0}, \mathbb{I})$$



$$\bar{x} = f_N \circ f_{N-1} \circ \dots \circ f_1(\bar{z})$$

$$\bar{y} = \bar{f}(\bar{z}) \Rightarrow p(\bar{y}) = p(\bar{f}(\bar{z})) = q(\bar{z}) \cdot \left| \det \frac{d\bar{z}}{d\bar{y}} \right|$$

$$\bar{z} \sim q(\bar{z}) \Rightarrow f \sim p(f(\bar{z})) = q(\bar{z}) \cdot \left| \det \frac{df}{d\bar{z}} \right|^{-1}$$

$$\log p(f(\bar{z})) = \log q(\bar{z}) - \log \left| \det \frac{df}{d\bar{z}} \right|$$

$$\frac{df}{d\bar{z}} = \frac{df_N}{d\bar{z}_{N-1}} \cdot \frac{df_{N-1}}{d\bar{z}_{N-2}} \cdot \dots \cdot \frac{df_1}{d\bar{z}}$$

$$\log p(f(\bar{z})) = \log q(\bar{z}_0) - \sum_{n=1}^N \log \left| \det \frac{df_n}{d\bar{z}_{n-1}} \right| \xrightarrow{f_n} \max$$

Autoregressive

$$\bar{f}(\bar{x}) = \bar{y}$$

$$(x_1, x_0) \quad (y_1, y_0)$$

$$y_i = f(x_1, \dots, x_i)$$

$$\det \frac{d\bar{y}}{d\bar{x}} = \prod_{i=1}^D \frac{dy_i}{dx_i}$$



RealNVP

$$(x_1, \dots, x_d, x_{d+1}, \dots, x_D) \mapsto (y_1, \dots, y_d, y_{d+1}, \dots, y_D)$$

$$\begin{pmatrix} \mathbb{I} & 0 \\ \text{lower triangular} & \text{diagonal} \end{pmatrix} \text{ d+k}$$

$$\bar{y}_{1:d} = \bar{x}_{1:d}$$

$$\bar{y}_{d+1:D} = \bar{x}_{d+1:D} \odot e^{\underline{s}(\bar{x}_{1:d})} + \underline{t}(\bar{x}_{1:d})$$

$$\bar{x}_{1-d} = \bar{y}_{1-d}$$

$$\bar{x}_{d+1-d} = (\bar{y}_{d+1-d} - \bar{\epsilon}(\bar{y}_{1-d})) \odot e^{s(\bar{y}_{1-d})}$$

MAF - Masked Autoregressive flows

$$p(\bar{x}) = \prod p(x_i | \bar{x}_{1:i-1})$$

$$\bar{z}_i \mapsto \bar{x} \quad p(x_i | \bar{x}_{1:i-1}, \bar{z})$$

$$x_i = \underbrace{z_i}_{\text{latent}} \underbrace{\sigma_i(\bar{x}_{1:i-1}) + \mu_i(\bar{x}_{1:i-1})}_{\text{masking}}$$

$$z_i = \frac{x_i - \mu_i(\bar{x}_{1:i-1})}{\sigma_i(\bar{x}_{1:i-1})}$$

$$\bar{z} = \frac{\bar{x} - \bar{\mu}(\bar{x})}{\bar{\sigma}(\bar{x})}$$

$$\frac{dx_i}{dz_i} = \sigma_i(\bar{x}_{1:i-1})$$

$$|\det \frac{d\bar{x}}{d\bar{z}}| = \prod_i \sigma_i(\bar{x}_{1:i-1})$$

$$f^{-1}: \bar{x} \mapsto \bar{z}, \quad d\bar{x}/d\bar{z} - \text{FAST}$$

$$f: \bar{z} \mapsto \bar{x} - \text{SLOW}$$

IAF - Inverse Autoregr. Flows

$$f: \bar{z} \mapsto \bar{x}$$

$$x_i \sim p(x_i | \bar{z}_{1:i-1}), \quad x_i = z_i \cdot \sigma_i(\bar{z}_{1:i-1}) + \mu_i(\bar{z}_{1:i-1})$$

$$\bar{x} = \bar{z} \odot \bar{\sigma}(\bar{z}) + \bar{\mu}(\bar{z})$$

$$z_i = \frac{x_i - \mu_i(\bar{z}_{1:i-1})}{\sigma_i(\bar{z}_{1:i-1})}$$

$$f^{-1}, d\bar{x}/d\bar{z} - \text{SLOW}$$

$$f: \bar{z} \mapsto \bar{x} - \text{FAST}$$