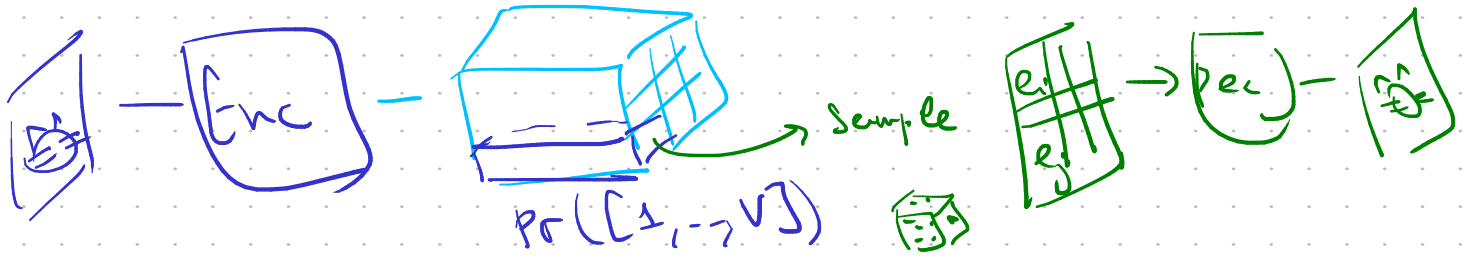
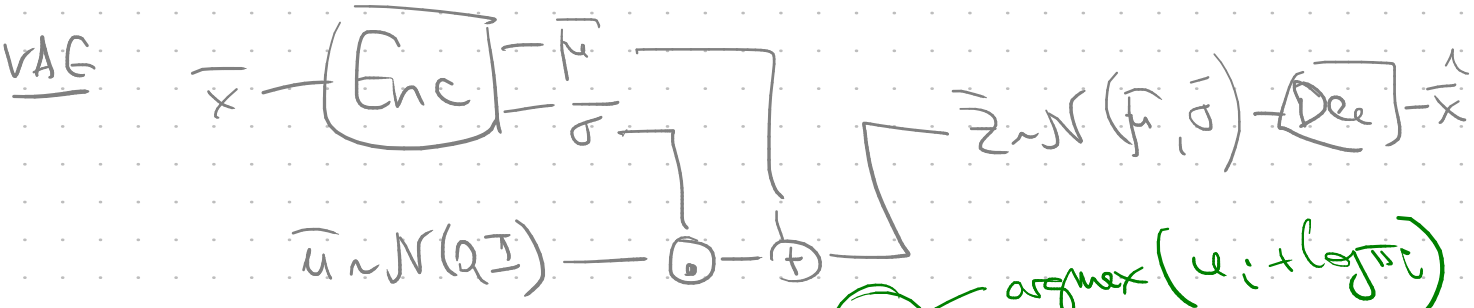
① VQ-VAE / VQ-GAN

② dVAE - distributional VAE / discrete VAE



VAE



$\text{argmax}(u_i + \log \pi_i)$

$\pi = (\pi_1, \dots, \pi_k)$

$u \sim$? Gumbel

Gumbel-Max trick

$g_i \sim \text{Gumbel}$, $g_i \in (-\infty, \infty)$

$p(g_i) = e^{-(g_i + e^{-g_i})}$

$F(g_i) = \int_{-\infty}^{g_i} e^{-(a+e^{-a})}\, da = e^{-e^{-g_i}}$

Thm. $z \sim \text{Mult}(\pi)$, $\pi = (\pi_1, \dots, \pi_k)$ $\Leftrightarrow$ $z = \underset{i}{\text{argmax}}(g_i + \log \pi_i)$ ye $g_i \sim \text{Gumbel}$

Proof. $p(z=k) = p(\forall j \; g_k + \log \pi_k \geq g_j + \log \pi_j) =$

$= \int_{-\infty}^{\infty} p(\forall j \neq k \; g_j + \log \pi_j \leq g_k + \log \pi_k) \cdot p(g_k)\, dg_k =$

$$= \int_{-\infty}^{\infty} \prod_{j \neq k} p\left(g_j + \log\pi_j \leq g_k + \log\pi_k\right) p(g_k)\, dg_k =$$

$$F(g_k + \log\pi_k - \log\pi_j)$$

$$= \int_{-\infty}^{\infty} \prod_{j \neq k} e^{-e^{-g_k - \log\pi_k + \log\pi_j}} \cdot e^{-(g_k + e^{-g_k})}\, dg_k =$$

$$\prod_{j \neq k} e^{-\pi_j} \cdot e^{-(g_k + \log\pi_k)} = e^{-\sum_{j \neq k} \pi_j} e^{-g_k + \log\pi_k}$$

$$= e^{1 - \pi_k}$$

$$= \int_{-\infty}^{\infty} e^{-e^{-(g_k + \log\pi_k)} \cdot \left(\sum_{j \neq k} \pi_j\right)} \cdot e^{-(g_k + e^{-g_k})}\, dg_k$$

$$= \int_{-\infty}^{\infty} e^{-e^{-(g_k + \log\pi_k)} \cdot (1 - \pi_k) - g_k - e^{-g_k}}\, dg_k =$$

$$\phi(g_k + \log\pi_k)$$

$$= \int_{-\infty}^{\infty} e^{-(g_k + \log\pi_k) - e^{-(g_k + \log\pi_k)} + g_k + \log\pi_k + \pi_k e^{-(g_k + \log\pi_k)} - g_k - e^{-g_k}}\, dg_k$$

$$= \int_{-\infty}^{\infty} e^{-(g_k + \log\pi_k) - e^{-(g_k + \log\pi_k)}} \cdot e^{\log\pi_k}\, dg_k = \pi_k$$

Gumbel-softmax trick

