1) Flow-based models

- Gen. models
  - Explicit
    - Autoregr.
    - Density-based
      - diffusion models
      - $\boxed{\text{flow-based}}$
  - Implicit
    - GAN

$$\bar{x} = f_N \circ f_{N-1} \circ \dots \circ f_1(\bar{z})$$

$$\bar{y} = \bar{f}(\bar{z}) \Rightarrow p(\bar{y}) = p(\bar{f}(\bar{z})) = q(\bar{z}) \cdot \left| \det \frac{d\bar{z}}{d\bar{y}} \right| =$$

$$\bar{z} \sim q(\bar{z}) \qquad = q(\bar{z}) \cdot \left| \det \frac{d\bar{y}}{d\bar{z}} \right|^{-1}$$

$$\bar{z} \sim q(\bar{z}) \rightsquigarrow \bar{f} = \bar{f}_N \circ \bar{f}_{N-1} \circ \dots \circ \bar{f}_2 \circ \bar{f}_1(\bar{z}) \rightsquigarrow$$

$$\rightsquigarrow \underline{p(\bar{x})} = p(\bar{f}(\bar{z})) = \underline{q(\bar{z})} \left| \det \frac{d\bar{f}}{d\bar{z}} \right|^{-1}$$

$$\frac{d\bar{f}}{d\bar{z}} = \frac{d\bar{f}_N}{d\bar{z}_{N-1}} \cdot \frac{d\bar{f}_{N-1}}{d\bar{z}_{N-2}} \cdot \ldots \cdot \frac{d\bar{f}_1}{d\bar{z}}$$

$$\bar{z} = \bar{z}_0 \to \bar{f}_1 \to \bar{z}_1 = \bar{f}_1(\bar{z}_0) \to \bar{f}_2 \to \dots \to \bar{z}_{N-1} = \bar{f}_{N-1}(\bar{z}_{N-2}) \to \bar{f}_N$$

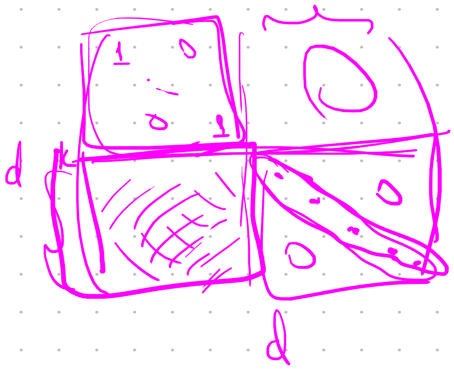
$$\boxed{\log p(\overset{\bar{x}}{\bar{f}(\bar{z})}) = \log q(\bar{z}) - \sum_{n=1}^{N} \log \left| \det \frac{d\bar{f}_n}{d\bar{z}_{n-1}} \right|} \to \max_{\bar{f}_1, \bar{f}_2, \dots, \bar{f}_N}$$

$\boxed{\bar{f}_n = ?}$

1) $\bar{f}_n$ обратима

2) $\det \frac{d\bar{f}_n}{d\bar{z}_{n-1}}$ легко вычислить

② Real NVP   real-valued non-volume preserving flow



$$\bar z = (z_1, z_2, \dots, z_k, z_{k+1}, \dots, z_d)$$

$$\bar y = (y_1 \dots y_k, y_{k+1}, \dots, y_d)$$

$$\bar y_{1:k} = \bar z_{1:k}$$

$$\bar y_{k+1:d} = \bar z_{k+1:d} \odot \sigma(\bar z_{1..k}) + \bar\mu(\bar z_{1..k})$$

$$e^{s(\bar z_{1..k})}$$

$$\det J = \prod_{j=k+1}^{d} e^{s_j(\bar z_{1..k})} = e^{\sum_j s_j(\bar z_{1..k})}$$

$$\bar z_{1..k} = \bar y_{1..k}, \quad \bar z_{k+1..d} = \frac{\bar y_{k+1..d} - \bar\mu(\bar y_{1..k})}{\sigma(\bar y_{1..k})}$$

③ Masked Autoregressive Flows (MAF)

$x_1 = z_1 \cdot \sigma_1 + \mu_1$

$x_2 = z_2 \cdot \sigma_2(x_1) + \mu_2(x_1)$

$= z_2 \cdot \sigma_2(z_1) + \mu_2(z_1)$

$\bar z \mapsto \bar x$ $p(\bar x) = \prod_i p(x_i | x_1 \dots x_{i-1})$

$p(x_i | \underbrace{x_1 \dots x_{i-1}}, \bar z)$

PixelCNN

$$p(x_i | \bar x_{1..i-1}, \bar z) = z_i \cdot \sigma_i(\bar x_{1..i-1}) + \mu_i(\bar x_{1..i-1}) =$$

$$= z_i \cdot f(\bar z_{1..i-1}) + g_i(\bar z_{1..i})$$

$$\frac{dx_i}{dz_i} = \sigma_i(\bar x_{1..i-1})$$

$$\boxed{z_i = \frac{x_i - \mu_i(\bar x_{1..i-1})}{\sigma_i(\bar x_{1..i-1})}}$$

$$\det \frac{d\bar x}{d\bar z} = \prod_i \sigma_i(\bar x_{1..i-1})$$

$f^{-1}: \bar x \mapsto \bar z$ — fast | $f: \bar z \mapsto \bar x$ — slow

$x_1, x_2, x_3 \dots$

4 Inverse Autoregressive Flows (IAF)

$$x_i \sim p(x_i \mid \bar z_{1..i})$$

$$x_i = z_i \cdot \sigma_i(\bar z_{1..i-1}) + \mu_i(\bar z_{1..i-1})$$

$$\frac{dx_i}{dz_i} = \sigma_i(\bar z_{1..i-1}) \qquad \underline{\bar x = \bar z \odot \bar\sigma(\bar z) + \bar\mu(\bar z)}$$

$\bar x \mapsto \bar z$:

$$z_1 = \frac{x_1 - \mu_1}{\sigma_1}$$

$$z_2 = \frac{x_2 - \mu_2(z_1)}{\sigma_2(z_1)} \quad \ldots \quad z_k = \frac{x_k - \mu_k(z_{1..k-1})}{\sigma_k(z_{1..k-1})}$$

$f: \bar z \mapsto \bar x$ – <u>fast</u> || $f^{-1}: \bar x \mapsto \bar z$ – <u>slow</u>