

① Flow matching

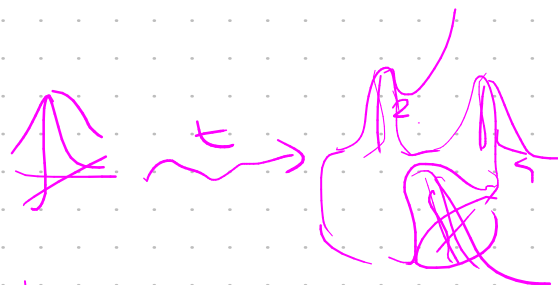
$$\bar{x}_0 \sim p_0, \bar{x}_k = f_{k0} \dots \circ f_k(\bar{x}_0)$$

$$\log p_k(\bar{x}_k) = \log p_0(\bar{x}_0) - \sum_k \log \left| \det \frac{\partial f_k}{\partial \bar{x}_{k-1}} \right|$$

RNF (residual NF)

$$\bar{x}_{k+1} = \bar{x}_k + \underline{\varepsilon} \cdot g_{\bar{\theta}}(\bar{x}_k, t_k)$$

$$\frac{\bar{x}_{k+1} - \bar{x}_k}{\varepsilon} = g_{\bar{\theta}}(\bar{x}_k, t_k)$$



CNF (continuous NF)

$$\bar{x} : \bar{x}_0 \xrightarrow{t} \bar{x}_1$$

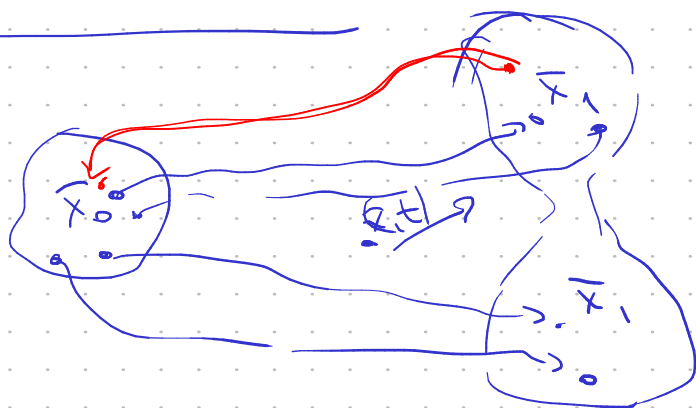
$$\frac{d\bar{x}_t}{dt} = v_{\bar{\theta}}(\bar{x}_t, t), \quad t \in [0, 1], \quad \bar{x}_0 \sim p_0, \quad \bar{x}_1 \sim p_1$$

$$\frac{d \log p_t(\bar{x}_t)}{dt} = -\operatorname{div} v_{\bar{\theta}} = -\sum_i \frac{\partial v_i}{\partial x_i} = -\operatorname{Tr} \left(\frac{dv_{\bar{\theta}}}{d\bar{x}} \right)$$

$$\log p_1(\bar{x}_1) = \log p_0(\bar{x}_0) - \int_0^1 \operatorname{Tr} \left(\frac{dv_{\bar{\theta}}}{d\bar{x}} \right) dt$$

rev. diffusion :

$$\nabla_{\bar{x}} \log p_t(\bar{x}) - \text{score function}$$



Lagrangian : $\bar{x}_t = \bar{x}(t)$

Eulerian : $v_t(\bar{x}) = v(\bar{x}, t)$

$$v_{\bar{\theta}}(\bar{x}, t) = ?$$

$$\frac{\partial p_t(\bar{x})}{\partial t} + \operatorname{div}(p_t(\bar{x}) \cdot v_t(\bar{x})) = 0$$

- sample $\bar{x}_0 \sim p_0, \bar{x}_1 \sim p_1 (\sim D), t \sim \text{Unit}([0,1])$



$$\bar{x}_t = \bar{\Psi}_t(\bar{x}_0, \bar{x}_1) = (1-t)\bar{x}_0 + t\bar{x}_1$$

$$(\bar{x}_1 - \bar{x}_0) = \mathbb{E} \left[\frac{\partial \bar{\Psi}_t(t)}{\partial t} \right] = \sigma_{\bar{\theta}}(\bar{x}_t, t)$$

$$h(\bar{\theta}) = \mathbb{E}_{\bar{x}_0, \bar{x}_1, t} \left[\left\| \sigma_{\bar{\theta}}(\bar{x}_t, t) - \frac{\partial \bar{\Psi}_t}{\partial t} \right\|^2 \right]$$

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