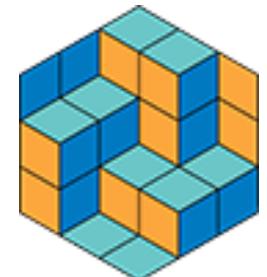
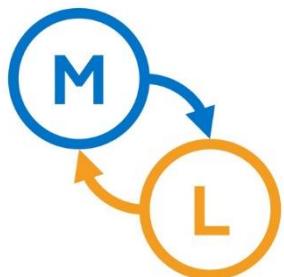


# Why Diffusion Models Don't Memorize: The Role of Implicit Dynamical Regularization in Training

Bonnaire, Urfin, Biroli, Mézard  
NeurIPS 2025



Brown: observes Brownian motion  $\mathbf{B}(t)$

Laplace, Fourier: heat  $H'(t) = \Delta H(t)$

Einstein: Avogadro number via BM

Fokker Planck equation, Langevin dynamics

Wiener: BM as a Fourier series

Polya: BM as a limit of Random Walk

Ornstein-Uhlenbeck  $dx(t) = -x(t)dt + B(t)$

Itô: Stochastic DEs  $dt = (dB(t))^2$

Feynman-Kac path integrals

Hairer: renormalization for rough paths

**Non-equilibrium thermodynamics**

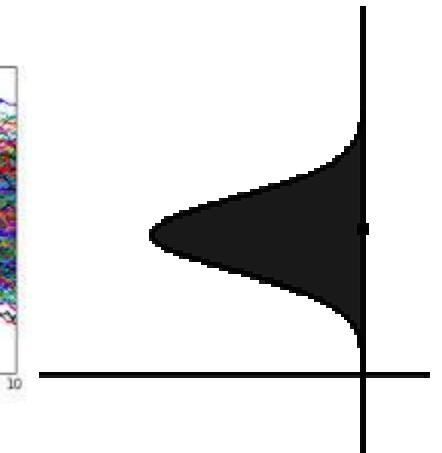
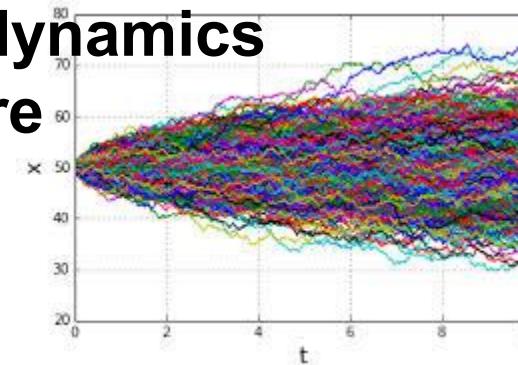
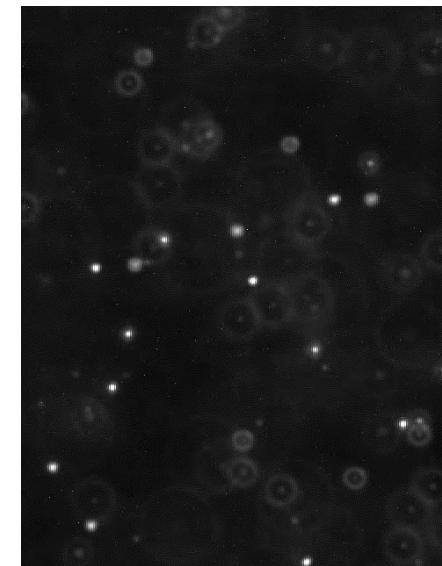
**Diffusions are everywhere**

**BM/RW is universal**

**Trajectories vs flows**

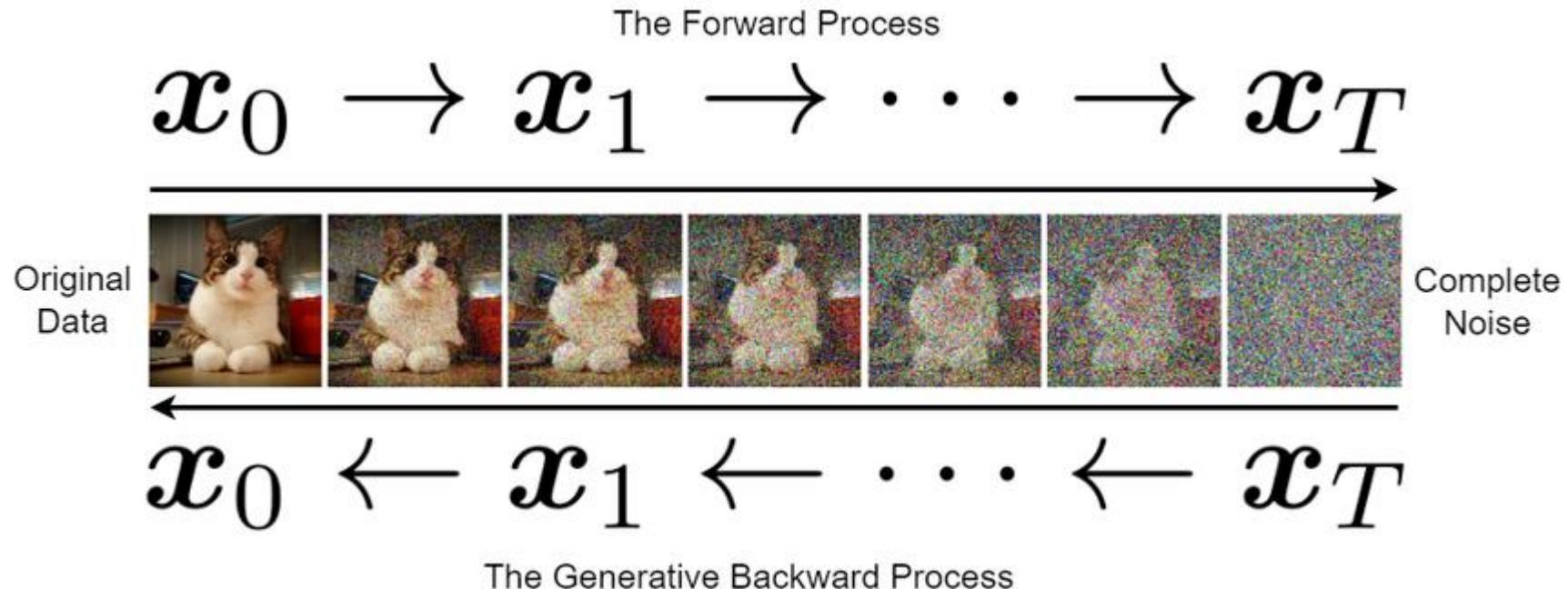
**Fourier analysis**

**?? Rough paths ??**

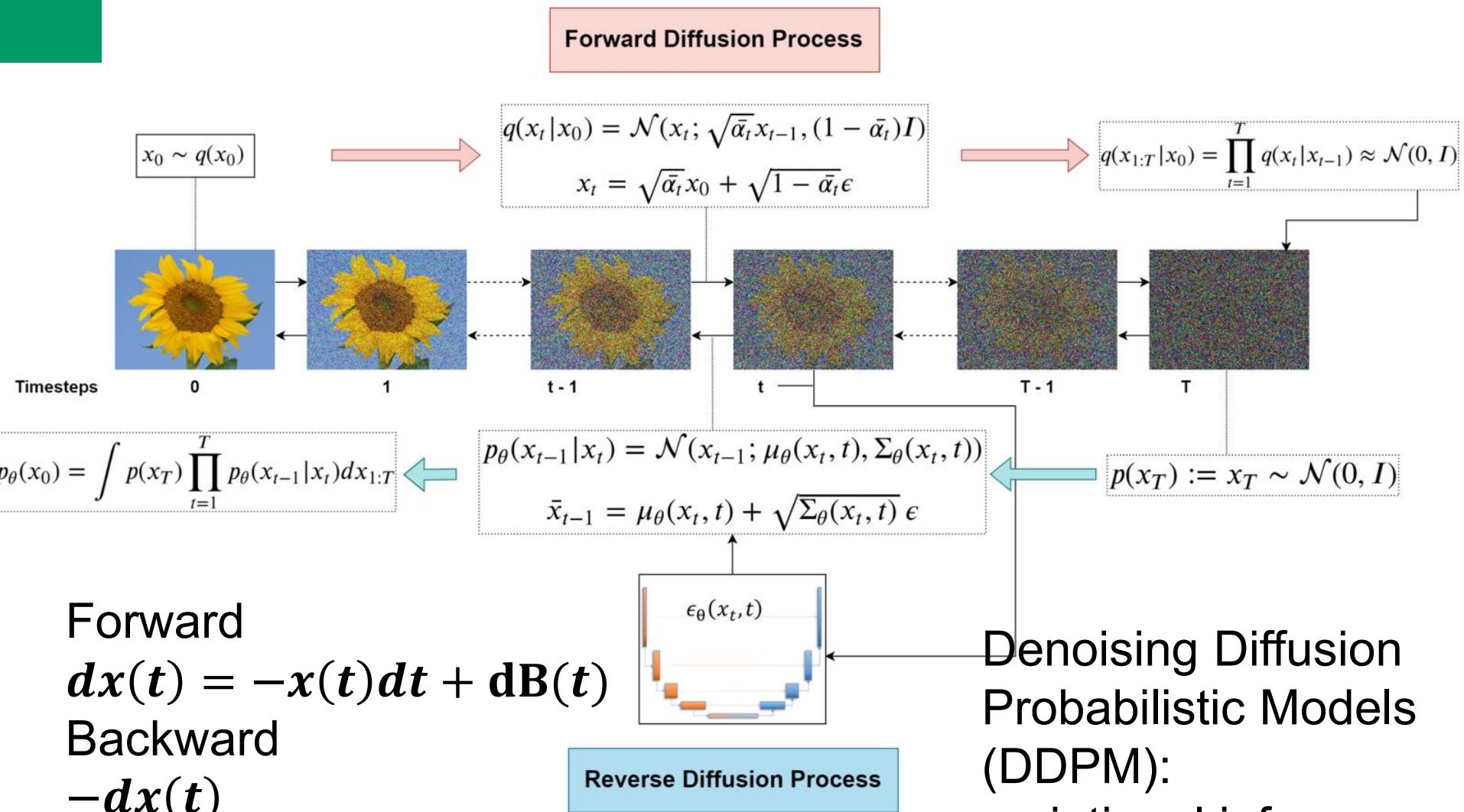


# Diffusion models

- **Sohl-Dickstein, Weiss, Maheswaranathan, Ganguli,(2015)**  
*Deep Unsupervised Learning using Nonequilibrium Thermodynamics*
- **Ho, Jain, Abbeel,(2020).**  
*Denoising Diffusion Probabilistic Models*
- **Song, Sohl-Dickstein, Kingma, Kumar, Ermon, Poole (2021)**  
*Score-Based Generative Modeling through Stochastic Differential Equations*
- Diffuse by Gaussian to get (almost) Boltzmann
- Integrate reverseSDE, try to match the score



# Diffusion models



# Pluses and minuses

- + **Training stability** No adversarial min–max optimization
- + **High sample quality** Excellent mode coverage Low artifacts
- + **Flexibility** Conditional generation Inpainting, super-resolution, editing
- + **Strong theoretical grounding** Explicit likelihood Well-defined SDE
- **Sampling cost** Requires tens to hundreds of denoising steps
- **Compute-intensive training** Large models, long training times
- **Less interpretable representations** Cf. VAEs / latent-variables



## Why Diffusion Models Don't Memorize: The Role of Implicit Dynamical Regularization in Training

[Tony Bonnaire](#), [Raphaël Urfin](#), [Giulio Biroli](#), [Marc Mezard](#)

Implicit dynamical regularization during training gives diffusion models a generalization window that widens with the training set size, so stopping within this window prevents memorization.

Why don't diffusion models memorize training data?

In principle they could – overparametrization!

Yet, empirically, diffusion models generalize extremely well, generate novel samples, interpolate smoothly, **but** memorization only appears very late in training, if at all.

Main claim: that generalization in diffusion models is driven primarily by training dynamics, through a form of implicit dynamical regularization.

# Empirical observations

$32 \times 32$  portraits,  $p = 4 \times 10^6$  trainable parameters  
 FID = 2-Gaussian Earth Mover Distance to training set  
 $f_{mem}$  = distance to the net of samples

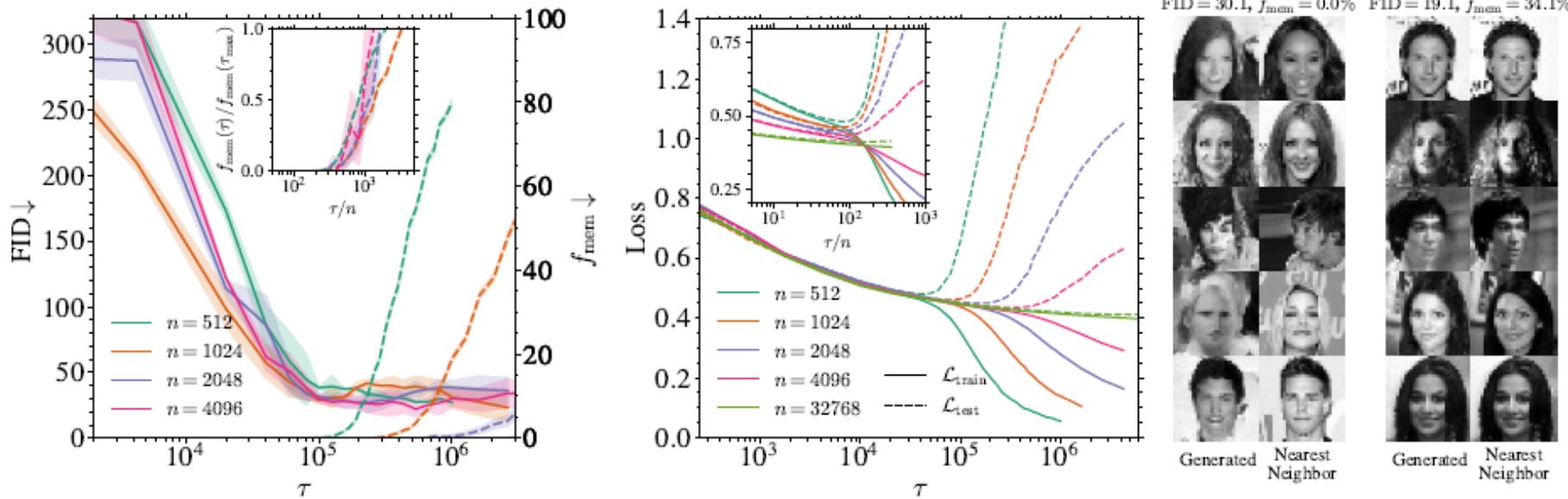


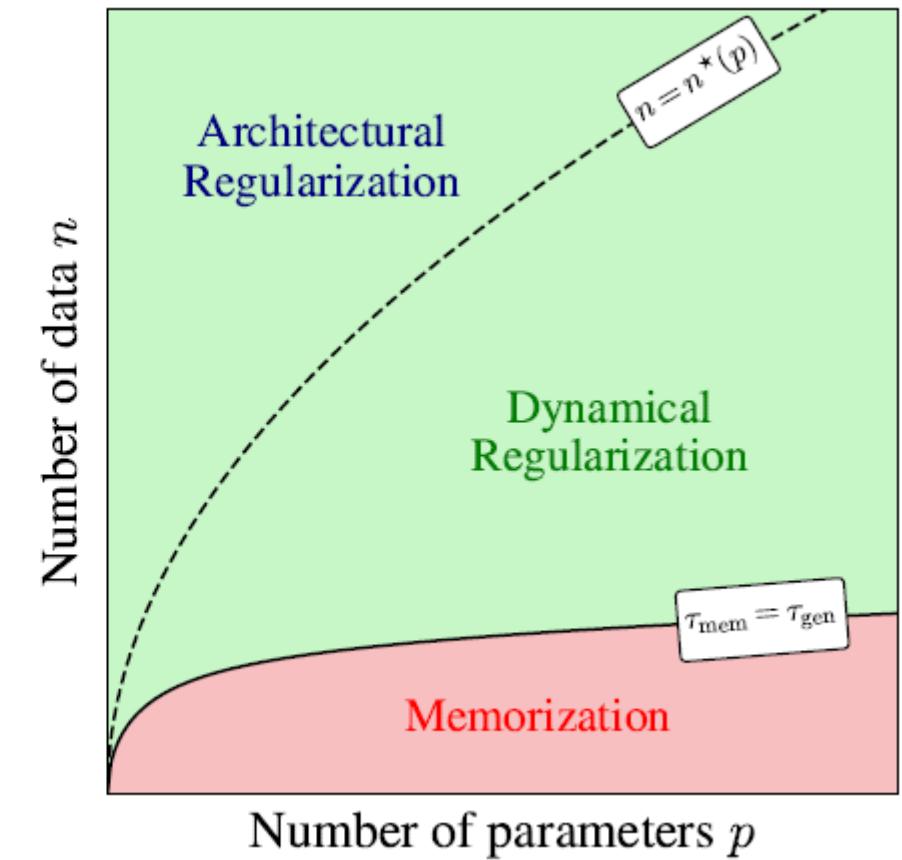
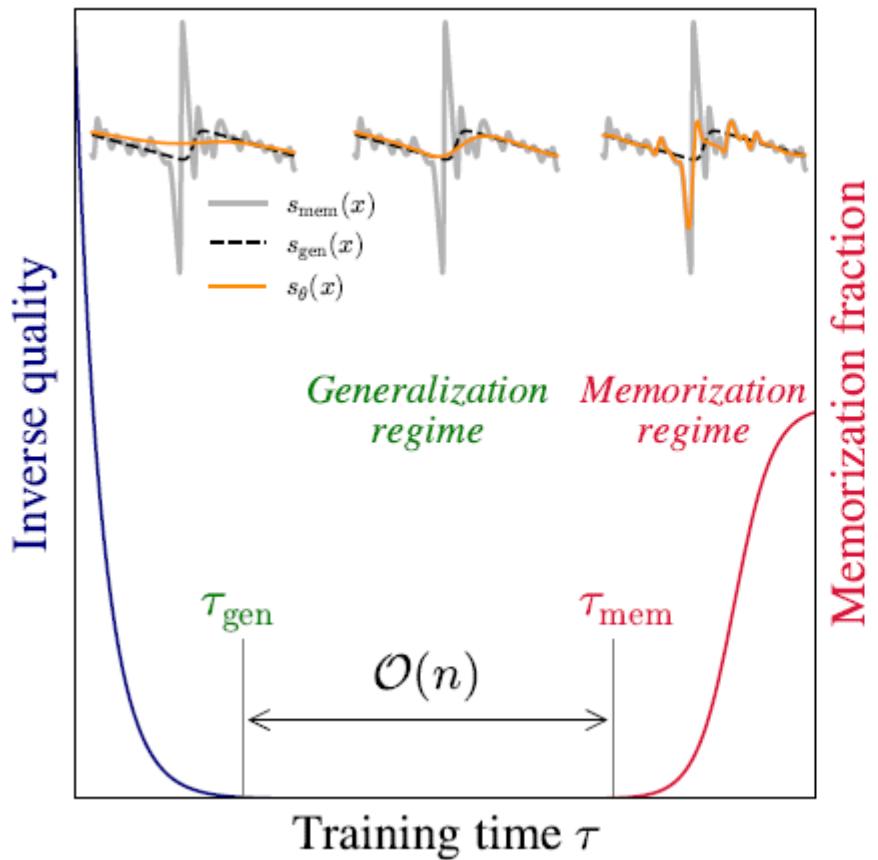
Figure 2: **Memorization transition as a function of the training set size  $n$  for U-Net score models on CelebA.** (Left) FID (solid lines, left axis) and memorization fraction  $f_{mem}$  (dashed lines, right axis) against training time  $\tau$  for various  $n$ . Inset: normalized memorization fraction  $f_{mem}(\tau)/f_{mem}(\tau_{max})$  with the rescaled time  $\tau/n$ . (Middle) Training (solid lines) and test (dashed lines) loss with  $\tau$  for several  $n$  at fixed  $t = 0.01$ . Inset: both losses plotted against  $\tau/n$ . Error bars on the losses are imperceptible. (Right) Generated samples from the model trained with  $n = 1024$  for  $\tau = 100K$  or  $\tau = 1.62M$  steps, along with their nearest neighbors in the training set.

# Empirical observations

$\tau_{gen} \approx \text{const}$ : onset of good sample quality

$\tau_{mem} \asymp \text{dataset size}$ : onset of memorization

Hence growing window without memorization



# Theoretic rationale

Memorization is ultimately driven by the overfitting of the empirical score.

Initially  $L_{train}$  and  $L_{test}$  are indistinguishable, but beyond a critical time,  $L_{train}$  continues to decrease while  $L_{test}$  increases, with generalization loss depending on  $n$ .

Memorization is not due to data repetition –even if at fixed  $t$  all models have processed each sample equally often, larger  $n$  postpone memorization.

Instead, we see **implicit dynamical regularization**: regularization arises indirectly from the **optimization dynamics themselves**, via **spectral bias**.

Smooth, low-frequency components are learned quickly. Highly oscillatory, high-frequency components are learned slowly.

# Toy model for analysis

Score: **linear random-features model**  $s(x) = \sum_k w_k \phi_k(x)$

- Train **full-batch gradient descent** on the denoising
- Features  $\phi_k$  are fixed; only weights  $w_k$  are trained
- Data drawn i.i.d. from a population distribution  $p_0$

**Training dynamics is exactly solvable**

- Gradient flow reduces to **linear regression**
- Diagonalize dynamics in the eigenbasis of the feature covariance  $C = \mathbf{E}[\phi(x)\phi(x)^T]$ , with eigenvalues  $\lambda_k$

**Closed-form solution**

Modes  $k$  evolve independently:  $w_{k(t)} = w_k(1 - e^{-\lambda_k t})$

**Conclusion**

Large  $\lambda_k$  (smooth) modes learned fast  $\rightarrow$  generalization

Small  $\lambda_k$  (fine-scale) modes learned slow  $\rightarrow$  delayed memorization

- Explains robustness of diffusion models
- Early stopping is theoretically justified
- Generalization driven by dynamics

## **Do we want to study DM at LM?**

- + A lot of SDE has not been used yet
- Need big compute?

## **Some things to do**

- Modify diffusion component
- Control high frequencies with Fourier
- Try to project on training set directly